Performance analysis on network connectivity for vehicular ad hoc networks

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Abstract: This paper proposes a simplified but reasonable uninterrupted highway model to evaluate the network connectivity performance for vehicular ad hoc networks (VANETs) from the view of both individual vehicle and global network. Three parameters: conditional connectivity of the communication pair of individual vehicles, global network connectivity probability, and the vehicle isolation probability are investigated. By combining the probability density function of inter-vehicle initial distance and the distribution of vehicles' relative speed, which are both derived in this paper, the closed-form of conditional connectivity is obtained. To derive the closed-forms of network connectivity and vehicle isolation probabilities, the analytical model takes into consideration the key factors such as communication range, vehicle speed, and enter intensity. The analytical results are validated by extensive simulations. Our derived highway network connectivity model could be applied in the study of a number of metrics related to connectivity in VANETs.

Keywords: VANETs; vehicular ad hoc networks; connectivity performance; isolation probability; vehicular communications.

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1 Introduction

Over the past decades, the widespread availability of wireless communication and handheld devices has stimulated the research on self-organising networks. These ad hoc networks consist of autonomous nodes that could collaborate in order to exchange information (Zhou and Haas, 1999), and have become increasingly popular (Royer and Toh, 1999; Mauve et al., 2001). One research issue that has drawn a lot of attention recently is the design of mobile ad hoc networks (MANETs). MANETs, which can be established without the aid of any fixed infrastructure, are comprised of nodes with computation and storage capabilities.

As a special case of MANETs, vehicular ad hoc networks (VANETs) are formed with self-organising vehicles which communicate through multi-hop wireless links in two communication modes as vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) (Mecklenbrauker et al., 2011; Molisch et al., 2009). An international standard for vehicular communication named wireless access in vehicular environments (WAVE) (based on the IEEE 802.11p physical-layer standard) was recently published (Amendment 6: Wireless Access in Vehicular Environments, 2010).

VANETs have attracted widespread attention and been supposed to play an important role in road safety and user applications to significantly decrease the number of road accidents and provide drivers with information and entertainment services during their journeys (Toor et al., 2008). The implement of VANETs could enable the rapid development of the next-generation Intelligent Transportation System to more efficiently accomplish the dissemination of safety and user information.

Several characteristics could distinguish VANETs from other types of MANETs, for example, the communication terminals in VANETs have no power constraints while their motions are restricted to the road patterns, and the random high mobility of network terminals may result in partitioned network segments. Due to the partitioned network, successful information propagation may fail to be guaranteed.

On account of the dynamic topology caused by high mobility, the network connectivity is always seriously affected and may not able to be guaranteed all the time. This will lead to multiple re-transmissions, extended transmission delay, and reduced network throughput (Khokhar et al., 2012). These performance deteriorations of inter-vehicle communication are not acceptable for the emergency and information data requested by the VANETs users (Khabazian et al., 2008). Hence, the network connectivity can be regarded as a fundamental technology issue and main performance measurement, and it is necessary to study the properties of network connectivity under the VANETs scenario specifically.

1.1 Related works

There have been numerous works about network connectivity published in the literature in recent years. A large proportion of existing results on the network connectivity were conducted for the two-dimensional MANETs (Bettstetter et al., 2005; Durrani et al., 2008; Yi et al., 2009; Lifang et al., 2009): Bettstetter et al. evaluated the effect of node density on the connectivity of wireless static networks in the shadow fading environment (Bettstetter et al., 2005). Based on the analytical results of Bettstetter et al. (2005), the authors of Durrani et al. (2008) analysed the impact of random beamforming on the connectivity in the presence of path loss and shadowing. Literature (Yi et al., 2009) defined three lifetime metrics to measure network stability under random mobility in the hierarchical MANETs. The path dynamics for MANETs were addressed in Lifang et al. (2009) by using a semi-markov smooth mobility model. In Jang et al. (2012), the authors proposed efficient strategies to maximise the connectivity for the cooperative communication in MANETs.

Since the nodes in MANETs always move with random directions while the tracks of vehicles are restricted along the road in VANETs, the analytical results of connectivity in MANETs cannot be straightforwardly applied into the analysis of VANETs. Therefore, a large number of analytical models were built to study the connectivity performance for one-dimensional ad hoc networks.

The literatures investigated the theoretical analysis of connectivity properties for one-dimensional ad hoc networks using stochastic model and queueing theory as well as percolation theory. Some investments focused on the k-connectivity (Wan et al., 2010), partial connectivity (Haiyan et al., 2010) or asymptotic connectivity (Honghai et al., 2005). The measurements of the connectivity performance includes the critical communication range, the distribution of the connectivity distance and node population, as well as the connectivity probability.

Fundamental characteristics of the connectivity in VANETs were studied in Shioda et al. (2008) with extensive simulations, and the authors addressed how the road and vehicle positions would affect the connectivity characteristic. A new mobility model was presented in Hafeez et al. (2010) to derive the number of vehicles on the road and connectivity probability. Two probabilities, the access probability that a vehicle can access the infrastructure and the connectivity probability that all the vehicles can connect to the infrastructure, were derived for vehicular relay networks in Seh et al. (2011). Reference Yousefi et al. (2008) studied the statistical properties of connectivity in VANETs with the queuing theory.

The authors in Cheng et al. (2012) discussed the impacts of the intervehicle spacing distribution on the connectivity of a vehicular ad hoc network in a highway traffic scenario based on a new empirical analysis. Reference Zhongjiang et al. (2012) studied the connectivity in VANETs based on the uniform stationary distribution without considering the mobility. A new mobility model was proposed in Hafeez et al. (2013) and the authors considered how the vehicles speeds would affect the connectivity and the packet reception rates. In Pitsiladis et al. (2012), spanning tree theory was adopted to present the connectivity calculation in finite wireless multi-hop networks. A general close form expression for the probability of the existence of at least one connected spanning tree inside the network's graph was derived. Literature (Ren et al., 2011) evaluated the percolation-based connectivity performance for the secondary network in a large-scale ad hoc heterogeneous network, based on the theory of continuum percolation.

Through analysing the existing research results, it is clear that most of the previous literatures studied connectivity in VANETs mainly from the macroscopic view, such as node population and cluster size in the road. The effect of individual vehicle's movement on connectivity, which can be regarded as the microscopic view, is still not clear. Macroscopic view can be considered as the viewpoint of the entire network and it mainly focuses on the global properties of the network, while microscopic view can be regarded as the viewpoint of a single vehicle or a communication pair of individual vehicles. In the viewpoint of the macroscopic, connectivity means the whole network is fully connected. In the viewpoint of microscopic, connectivity means that there exist some available paths between the communication pair of vehicles.

Since a great many safety-related services could be broadcasted only within a small-range zone rather than the whole network (for instance, the safety messages containing the lane-changing information or collision avoidance warning could be delivered to the surrounding vehicles only), the microscopic properties are of great significance to ensure the reliable safety-related communication in these cases. Moreover, the inter-vehicle distance is one of the key factors that affect the microscopic connectivity properties, which is impacted by the relative movements of the individual vehicles. Nonetheless, the distributions of the relative movement and inter-vehicle distance have been paid less attention in most of the previous works. The previous works analyse the connectivity properties considering either the vehicle mobility or the spatial distribution, while in this paper we take both the relative speed and inter-vehicle distance into account.

1.2 Contribution

The objective of this paper is to study the network connectivity in a highway mobility model from both the microscopic and macroscopic view. We focus on the distributions of the relative speed and inter-vehicle distance, and study how the individual vehicle's relative movements affect the microscopic and the macroscopic connectivity performance. We then employ three parameters: conditional connectivity probability, global network connectivity probability, and the vehicle isolation probability, to investigate the impact of the different system parameters on the network connectivity.

The conditional connectivity is the probability that a communication pair can stay connected during a whole data transmission conditioned on the connection is successfully built up when the transmission is initialised. In the VANETs scenario, various types of data are transmitted in the network. Some small sized data can be transmitted by packaging into a few or even one packet (for example, the emergency broadcast), while some other data may need much longer transmission duration (for example, the video-on-demand data stream). It is clear that the small sized data has better adaptivity to the not guaranteed network connectivity, but the transmission of data for entertainment services may face frequent link interruptions. Therefore, we will focus on the effect of conditional connectivity on the data transmission service like entertainment or other large-data-based ones. In particular, we will study how the conditional connectivity would be influenced by vehicles' relative mobility and inter-vehicle initial distance.

Moreover, different from the previous work (Chen et al., 2012) which only focuses on a communication pair of individual vehicles, this paper also studies the global properties of the whole network. Global network connectivity is the probability that the whole network is fully connected and an arbitrary vehicle can directly communicate with its neighbourhood; while vehicle isolation probability means the probability that a randomly chosen vehicle has no neighbour and cannot directly communicate with others. Considering a highly dynamic network topology and the stringent delay requirements for some of the VANETs safety-related applications, the delay-critical messages like collision avoidance warning are supposed to be disseminated immediately through direct one-hop communication. There should exist the fully connected paths for the safety-related message dissemination through the entire network to guarantee the successful reception of the whole network. In order to ensure the reliable broadcast for the one-hop safety-critical services, it is essential that there exists no isolated vehicle in the network and any arbitrary vehicle can establish the direct link with its neighbours. Hence, we will also study the impacts of network connectivity and vehicle isolation on the transmission of one-hop safety-related services.

Hence, these three parameters could reflect the connectivity properties in different application scenarios for VANETs. The conditional connectivity indicates how the vehicles' relative mobility and inter-vehicle initial distance impact the data transmission service like entertainment or other large-data-based ones. Network connectivity probability measures the successful delivery for the delay-critical messages dissemination through the entire network. The complement of vehicle isolation probability evaluates the reliability of the broadcast for one-hop safety-critical services.

An analytical model, which takes into account the vehicle enter intensity, variation range of the vehicles speed, communication range, and data transmission duration, is developed to study the factors that mainly influence the conditional connectivity probability, global network connectivity probability, and the vehicle isolation probability:

- we theoretically derive the distribution of inter-vehicle initial distance, which represents the interval of two vehicles that arrive at the highway entry in succession.
- we calculate the distribution of vehicles' relative speed and discuss the impact of relative speed on the probability of successful reception
- the closed-form of conditional connectivity probability is obtained
- network connectivity probability and the vehicle isolation probability are derived from the view of global network, based on the distributions of inter-vehicle distance and vehicle population size.

The analytical results of this paper are not restricted to a communication pair of individual vehicles, but also reflect the global properties of the whole network. Analytical results can be used to reflect the reliability performance of inter-vehicle communication in the highway scenarios, and help to design an effective vehicular network. Moreover, the results in the paper can provide the guidance to set appropriate system parameters in order to attain the required connectivity performance, and also enable drivers to select reasonable routes in the highway scenarios.

1.3 Outline

The remainder of the paper is organised as follows. The system model is briefly described in Section 2. In Section 3, the distributions of inter-vehicle initial distance and relative speed are derived, and the analysis of the connectivity performance is presented. In this section, the closed-forms of conditional connectivity probability, global network connectivity probability, and the vehicle isolation probability are obtained. Both analytical and simulation results are revealed and compared in Section 4. Finally, Section 5 concludes this paper.

2 System model

In this paper, some assumptions are illustrated to establish a simplified yet rational model for the unidirectional uninterrupted highway scenarios, which are given as follows.

- Each vehicle on the highway, with the length L, is assumed to have an equal communication range of R. According to unit disk model (Ng et al., 2011), which is adopted in this paper as the channel model, two vehicles can directly communicate with each other only if their Euclidean Distance is less than or equal to R.
- The vehicles enter the highway by following a Poisson process with an intensity of λ (i.e., the amount of the vehicles entering the highway per second). After arriving at the highway entry, each vehicle i is assigned with a random speed v_i, which is independent uniformly distributed in the interval of $[v_{\min}, v_{\max}]$. Then the speed of each vehicle does not change during its movement in the highway. The probability density function (pdf) of speed v_i can be given as:

$$f_{v_i}(x) = \begin{cases} \dfrac{1}{v_{\max} - v_{\min}} & v_{\min} \le x \le v_{\max} \\ 0 & \text{otherwise} \end{cases}. \quad (1)$$

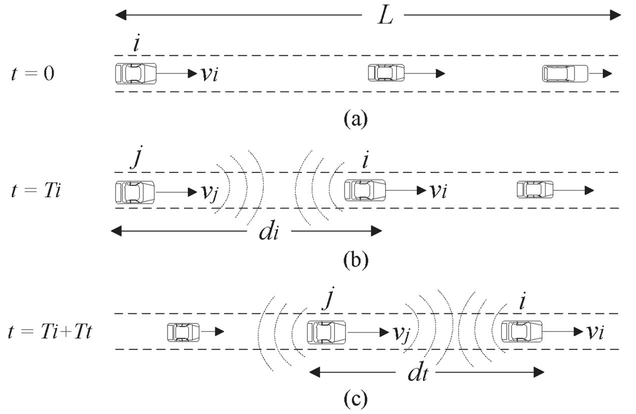
  Let random variable T_i represent the enter time interval between vehicle i and the subsequent vehicle j. According to the stochastic process theory, the enter time interval is independent identically distributed (i.i.d.) with exponential distribution with rate parameter λ. Then the pdf of T_i can be expressed as

$$f_{T_i}(y) = \begin{cases} \lambda \mathrm{e}^{-\lambda y} & y \ge 0 \\ 0 & y < 0 \end{cases}. \quad (2)$$

- To simplify the discussion, it is assumed that the movements of all vehicles are independent. Vehicles travelling on the unidirectional highway model are allowed to pass each other. Even though the vehicle passings are severely restricted by the number of lanes on the highway in real scenarios, there are no overtaking limitations involved in this paper in order to simplify the analytical study.

The unidirectional highway model used for this paper is demonstrated in Figure 1 to study the communication connectivity of the vehicles. As illustrated in Figure 1, two vehicles arrive at the highway entry in succession and begin to transmit the data stream. The speeds of vehicles will not change during their movements on the highway of length L. At the beginning of the observation, which is denoted as $t = 0$, a vehicle i arrives at the entry and begins moving along the road with the speed v_i, as revealed in Figure 1(a). After time interval T_i, a new vehicle j arrives with the speed v_j following vehicle i, as depicted in Figure 1(b). At this moment, the distance between i and j can be calculated as $d_i = v_i T_i$, which is defined as the inter-vehicle initial distance. Assume that vehicles i and j start transmitting data at the time $t = T_i$. Let T_t represent the duration of data transmission. If the link between i and j stays available during the whole transmission, then the whole packet can be successfully received at the time $t = T_i + T_t$. Figure 1(c) shows the movements of vehicle i and j and the distance between them which denotes d_t, as the data stream transmission is finished at $t = T_i + T_t$.

Figure 1 Simplified unidirectional highway model



3 Performance analysis

In order to make sure that the link between vehicle i and j can maintain accessible during the transmission, the distance between them should be kept no more than the communication range R. Therefore, the successful reception is closely associated with the inter-vehicle distance and the relative speed of the vehicles.

Firstly, the probability density function of inter-vehicle initial distance d_i can be derived. Secondly, the probability distribution of vehicles' relative speed is given and the distance d_t of the two vehicles is analysed by the end of the transmission. Next, the probability of successful reception is calculated as conditional connectivity probability. Finally, the network connectivity and the vehicle isolation probability are studied from the global viewpoint, based on the expectation of the vehicle population size.

3.1 Inter-vehicle initial distance

As shown in Figure 1, at the time $t = 0$, vehicle i arrives at the highway entry and continues moving with a randomly chosen speed v_i, which is uniformly distributed. After a time interval T_i, vehicle j arrives at the entry. At the moment $t = T_i$, the two vehicles are d_i apart, which is defined as the inter-vehicle initial distance. Then the inter-vehicle initial distance can be expressed as

$$d_i = v_i T_i, \tag{3}$$

where v_i and T_i are independent random variables. Therefore, their joint probability density function can be calculated as

$$f_{v_i,T_i}(x, y) = f_{v_i}(x) f_{T_i}(y). \tag{4}$$

By combining equations (1)–(3), the probability density function of inter-vehicle initial distance d_i can be calculated as

$$\begin{aligned} f_{d_i}(z) &= \int_{-\infty}^{\infty} \frac{1}{|y|} f_{v_i}\left(\frac{z}{y}\right) f_{T_i}(y)\mathrm{d}y \\ &= \frac{1}{v_{\max} - v_{\min}} \int_{z/v_{\max}}^{z/v_{\min}} \frac{\lambda}{y} \mathrm{e}^{-\lambda y}\mathrm{d}y \\ &= \frac{\lambda}{v_{\max} - v_{\min}} \int_{\lambda z/v_{\max}}^{\lambda z/v_{\min}} t^{-1}\mathrm{e}^{-t}\mathrm{d}t. \end{aligned} \tag{5}$$

The result of above integral could not be accurately calculated. Then the exponential integral $E_1(z)$ in Gradshteyn and Ryzhik (2007) can be used to resolve the pdf of d_i as:

$$\begin{aligned} f_{d_i}(z) &= \frac{\lambda}{v_{\max} - v_{\min}} \left(\int_{\frac{\lambda z}{v_{\max}}}^{\infty} t^{-1}\mathrm{e}^{-t}\mathrm{d}t - \int_{\frac{\lambda z}{v_{\min}}}^{\infty} t^{-1}\mathrm{e}^{-t}\mathrm{d}t \right) \\ &= \frac{\lambda}{v_{\max} - v_{\min}} \left[E_1\left(\frac{\lambda z}{v_{\max}}\right) - E_1\left(\frac{\lambda z}{v_{\min}}\right) \right] \end{aligned} \tag{6}$$

where

$$E_1(z) = \int_z^{\infty} \mathrm{e}^{-t} t^{-1}\mathrm{d}t. \tag{7}$$

Then the cumulative distributed function (CDF) of d_i can be given as

$$F(x) = \Pr(d_i \leq x) = \int_0^x f_{d_i}(z)\mathrm{d}z \quad , \tag{8}$$

which represents the probability that inter-vehicle initial distance d_i is no more than the value x. Hence, the probability that d_i is no more than communication range R, which indicates the probability that the two adjacent vehicles could communicate directly once they arrives at the highway entry in succession, can be expressed as:

$$\Pr(d_i \leq R) = \int_0^R f_{d_i}(z)\mathrm{d}z. \tag{9}$$

According to the result of formula (6), the distribution of inter-vehicle initial distance can be regarded as the difference between exponential integrals, which depends on the vehicle enter intensity λ and speed variation range $[v_{\min}, v_{\max}]$. In order to evaluate the impact of vehicle speed and enter intensity on inter-vehicle initial distance d_i, both analytical and simulation results of the CDF of inter-vehicle initial distance with different λ, $v_{\min}$ and $v_{\max}$ are separately illustrated in Figures 2–4. As shown in the figures, the analysis results match the simulation results, validating the correctness of theoretical derivation.

In Figure 2, $v_{\max}$ and $v_{\min}$ are fixed at 40 m/s and 20 m/s respectively, while λ changes from 0.25 to 1. As shown in the figure, the trends of curves ascend more apparently with larger λ. The probability that d_i is no more than 250 m is around 0.87 when λ is 0.25, while the probability can achieve almost 1 when λ is 0.75 or 1. This is because that higher enter intensity indicates larger vehicle density and shorter inter-vehicle distance, which significantly improve the connectivity probability.

Figure 2 CDF of d_i with various λ when vehicles move with higher speed

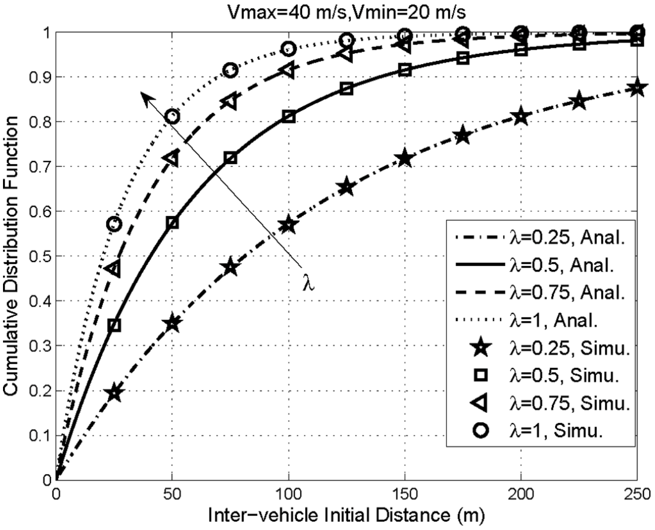


Figure 3 CDF of d_i with various λ when vehicles move with lower speed

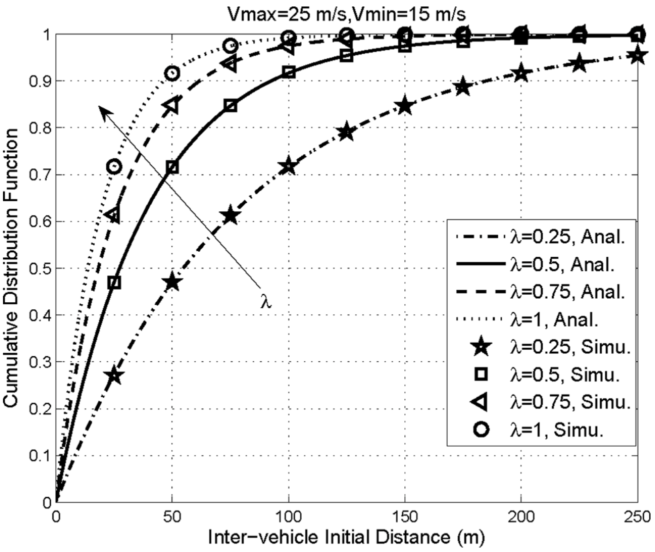
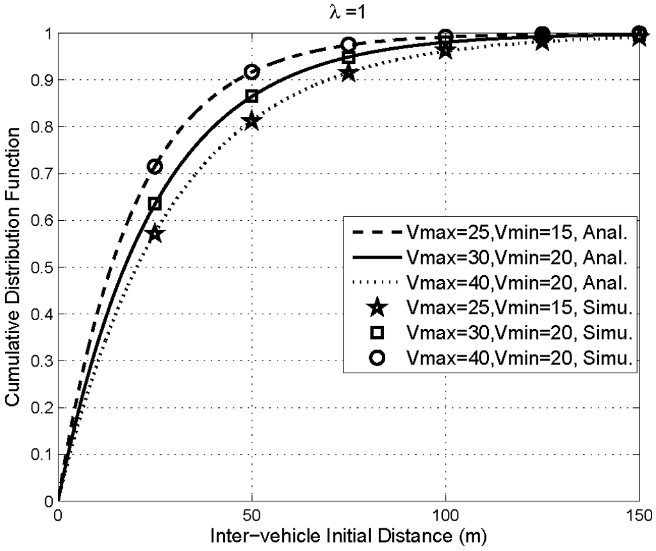


Figure 3 illustrates both the analytical and simulation results when $v_{\max}$ and $v_{\min}$ are set to 25 and 15 m/s separately. Obviously, the trends of the curves in the figure are the same as those in Figure 2. Compared to the Figure 2, it is clear that the curves shown in Figure 3 rapidly climb faster. The probability that d_i is no more than 50 m is about 0.8 when λ is 1 in Figure 2, while the probability can achieve more than 0.9 in Figure 3. Therefore, it can be drawn the conclusion that the slower vehicle speed leads to the closer inter-vehicle initial distance and then increases the connectivity performance.

In Figure 4, λ is fixed at 1 as the speed variation ranges $v_{\max}$ and $v_{\min}$ are varied. It can be seen that the vehicles with lower speed are more possible to stay close to each other rather than those with higher mobility. The reason is that the vehicles with higher speed could move a longer distance, which results in the larger inter-vehicle distance. It can be drawn a conclusion from the figure that larger enter intensity and lower speed can bring about shorter inter-vehicle initial distance.

Figure 4 CDF of d_i with different speed variation ranges when λ is fixed



3.2 *Conditional connectivity probability*

Conditional connectivity is the probability that a communication pair can stay connected during a whole data transmission conditioned on the connection is successfully built up when the transmission is initialised. The data transmission duration T_t is fixed and each vehicle's speed keeps constant during the transmission. Therefore, the conditional connectivity probability can be considered as the probability that the vehicles are still within the communication range of each other until the transmission is successfully finished.

Assume that the receiver is located within the range of transmitter at the beginning of the data stream transmission. Conditional connectivity probability is defined as the probability that the communication link between two consecutive vehicles can maintain connected during the whole transmission, given that the vehicles begin to transmit data stream once they arrive at the highway entry in succession.

In accordance to the highway model, vehicle i and j begin to communicate once j arrives at the entry at the moment $t = T_i$, and they are d_i apart initially. After transmission duration T_t, the two vehicles are still connected with the distance d_t, where

$$d_t = |d_i + (v_i - v_j)T_t| = |d_i + \Delta v T_t|. \tag{10}$$

Then the conditional connectivity probability P_{con} is the probability that the link between i and j could stay accessible during the whole data transmission, given that i and j are within the communication range of each other at the beginning of transmission.

$$P_{\text{con}} = \Pr(d_t \leq R \,|\, d_i \leq R) = \frac{\Pr(d_t \leq R, d_i \leq R)}{\Pr(d_i \leq R)}. \tag{11}$$

Therefore, the conditional connectivity probability P_{con} of two vehicles depends on their inter-vehicle initial distance d_i, relative speed $\Delta v = v_i - v_j$, the data transmission duration

T_t and communication range R. The pdf of relative speed Δv is given as follows:

$$f_{\Delta v}(u) = \begin{cases} \dfrac{u + v_{\max} - v_{\min}}{(v_{\max} - v_{\min})^2} & v_{\min} - v_{\max} \le u \le 0 \\ \dfrac{-u + v_{\max} - v_{\min}}{(v_{\max} - v_{\min})^2} & 0 < u \le v_{\max} - v_{\min} \\ 0 & \text{otherwise} \end{cases}. \tag{12}$$

In case $v_i \ge v_j$, that is $\Delta v \ge 0$ and $d_t = |d_i + \Delta v T_t| = d_i + \Delta v T_t$, then the distance between the two vehicles becomes increasingly growing. The joint probability p_1 of the events $\{d_t \le R\}$, $\{d_i \le R\}$ and $\{v_i \ge v_j\}$ can be calculated as:

$$\begin{aligned} p_1 &= \Pr(d_t \le R, d_i \le R, v_i \ge v_j) \\ &= \Pr(0 \le d_i \le R - \Delta v T_t, \Delta v \ge 0) \\ &= \int_0^{v_{\max}-v_{\min}} \int_0^{R-\Delta v T_t} f_{d_i}(z) f_{\Delta v}(u) \mathrm{d}z \mathrm{d}u. \end{aligned} \tag{13}$$

The above integral may be numerically evaluated by substituting equations (6) and (12). In the same way, if $v_i < v_j$, that is $\Delta v < 0$ and vehicle j is catching up with i, then the joint probability p_2 in this case can be derived as:

$$\begin{aligned} p_2 &= \Pr(d_t \le R, d_i \le R, v_i < v_j) \\ &= \int_{v_{\min}-v_{\max}}^{0} \int_0^{R} f_{d_i}(z) f_{\Delta v}(u) \mathrm{d}z \mathrm{d}u. \end{aligned} \tag{14}$$

Substituting equations (6) and (12), the above integral is determined that $p_2 = \Pr(d_i \le R)/2$. According to the law of total probability and combining equations (13) with equation (14), the joint probability of events $\{d_t \le R\}$ and $\{d_i \le R\}$ can be expressed as:

$$\Pr(d_t \le R, d_i \le R) = p_1 + p_2. \tag{15}$$

Finally, the conditional connectivity probability P_{con} can be numerically calculated by substituting equations (9) and (15) to formula (11). Evidently, the result could be useful in the selection and design for effective VANETs routing protocols.

3.3 *Network connectivity probability*

The network connectivity of VANETs is a multifaceted problem due to the uncertainty of the network topology. Whereas we previously studied the level of connectivity from the viewpoint of a single vehicle and its neighbourhood, this section investigates the connectivity of the whole vehicles from a global network point of view. To estimate the global network connectivity of the proposed model, we introduce a measure, called *network connectivity probability*.

This parameter is defined as a measure of a snapshot during which the whole network is connected. It shows the essential properties of a highway network topology when the relative positions of vehicles and states of the links change within a small time duration. More importantly, this metric evaluates the connectivity performance from the whole network view, which means a randomly chosen vehicle in the network can directly communicate with its neighbourhood.

According to equation (6), the distribution of inter-vehicle initial distance d_i depends on the vehicle enter intensity λ and speed distribution interval $[v_{\min}, v_{\max}]$. Then the expectation of d_i can be calculated as follows:

$$E_{d_i}(z) = \int_{-\infty}^{\infty} z f_{d_i}(z) \mathrm{d}z = \int_0^{\infty} z f_{d_i}(z) \mathrm{d}z. \tag{16}$$

The mean of vehicle density is defined as the reciprocal of the expectation of d_i, which represents the average number of the vehicles in per metre on the highway, expressed as:

$$\mu_{\text{veh}} = \frac{1}{E_{d_i}(z)} = \frac{1}{\int_0^{\infty} z f_{d_i}(z) \mathrm{d}z}. \tag{17}$$

Therefore, the vehicle population size, which is the total number of vehicles on the highway of length L, can be derived from $N_{\text{veh}} = L\mu_{\text{veh}}$. The probability that the two adjacent vehicles are able to communicate directly, i.e., the probability that d_i is no more than communication range R, can be expressed as

$$F_{d_i}(R) = \Pr(d_i \le R) = \int_0^R f_{d_i}(z) \mathrm{d}z. \tag{18}$$

Based on above analysis, the network connectivity probability, which is the probability that the whole network is fully connected and any arbitrary vehicle in the network could communicate directly with its adjacent vehicles, can be computed as follows:

$$P_{\text{net}} = F_{d_i}(R)^{N_{\text{veh}}-1} = \left(\int_0^R f_{d_i}(z) \mathrm{d}z \right)^{L\mu_{\text{veh}}-1}. \tag{19}$$

3.4 *Vehicle isolation probability*

The degree of a vehicle can be defined as the number of neighbours. If a vehicle has no neighbour at all, it is in the state of isolation. The probability that an arbitrary vehicle has no neighbour, which is named the vehicle isolation probability, is a fundamental local network property of VANETs. The vehicle isolation probability could be regarded as the complement of the connectivity probability, indicating the impact of node link failures on connectivity performance (Miorandi and Altman, 2006).

The vehicle isolation probability is the probability that a randomly chosen vehicle cannot be able to communicate with any other vehicles in the network. The network remains connected if there are no isolated vehicles. In our model, the vehicle isolation probability, which is the probability that the vehicle cannot communicate with others, can be expressed as:

$$P_{\text{iso}} = 1 - F_{d_i}(R) = 1 - \int_0^R f_{d_i}(z) \mathrm{d}z. \tag{20}$$

Both the network connectivity probability and the vehicle isolation probability can be regarded as the global measurements of connectivity performance, and can also reflect the fundamental properties of network connectivity from the macroscopic view. In the following, we present the numerical results of the derived models.

4 Numerical results

To evaluate the analytical results of conditional connectivity probability, network connectivity probability and vehicle isolation probability in Section 3, a simulation platform is established with Matlab for different scenarios by changing the communication range, enter intensity and vehicle speed separately. In order to reduce the randomisation, 10^5 random scenarios are generated and 10^5 simulations are run for every situation. Transmission duration T_t is set to 5 seconds which is long enough to receive the data streaming for user applications. Different scenarios can be generated by increasing the communication range, vehicle speed variation and enter intensity, as shown in Table 1. We consider an uninterrupted highway segment of 2000 m in length, and all the simulation results are obtained by changing communication range R (varies from 10 m to 800 m in total) and enter intensity λ (from 0.25 to 1 veh/sec). As shown in the figures, the analysis results are identical with the simulation results, indicating the correctness of theoretical derivation.

Table 1 Simulation parameters

Parameters	*Values*
Highway length	2000 m
Transmission duration	5 s
Communication range	[75 m, 300 m] for P_{con} [100 m, 800 m] for P_{net} [10 m, 210 m] for P_{iso}
Enter intensity	0.25, 0.5, 0.75, 1 veh/sec
Maximum vehicle speed	40 m/s, 25 m/s
Minimum vehicle speed	20 m/s, 15 m/s

4.1 Conditional connectivity probability

Figure 5 illustrates both the analytical and simulation results of conditional connectivity probability P_{con} with different communication range R and enter intensity λ when $v_{\max}$ and $v_{\min}$ are set to 40 and 20 m/s respectively. As shown in the figure, the simulation results approximate to the analytical results, which validates the correctness of theoretical derivation, while the gaps are mainly caused by the approximate calculation error of integral. It can be seen from the figure that P_{con} increases with raised communication range R, which means that the larger communication range, the better connectivity performance. Obviously, P_{con} with higher λ is larger than that with lower λ under the specific communication range. This can be explained that higher enter intensity leads to larger vehicle density, which makes the network dense and improves the connectivity probability. Therefore, it comes to a conclusion that the vehicles are more possible to stay connected during the whole transmission with the larger communication range and higher enter intensity, under the specific $v_{\max}$ and $v_{\min}$.

Figure 6 demonstrates the analytical and simulation results of conditional connectivity probability when $v_{\max}$ and $v_{\min}$ are fixed at 25 and 15 m/s separately. Apparently, the trends of curves in the figure are identical with those in Figure 5. The conditional connectivity probability P_{con} increases with the further communication range R and added enter intensity λ. In contrast with Figure 5, it is evident that the vehicles with lower average speed are more possible to successfully exchange information rather than those with higher speed. For instance, for a fixed communication range $R = 100$ m and a specific enter intensity $\lambda = 0.5$, the P_{con} is around 0.9 with the higher mobility in Figure 5 while the probability can achieves about 0.97 with slower speed in Figure 6. The reason is that network topology changes dynamically and rapidly due to high mobility. As a consequence, the links between the vehicles may be interrupted during the transmission, which leads to reduce the communication connectivity performance.

Figure 5 Conditional connectivity probability with various λ when vehicles move with higher speed

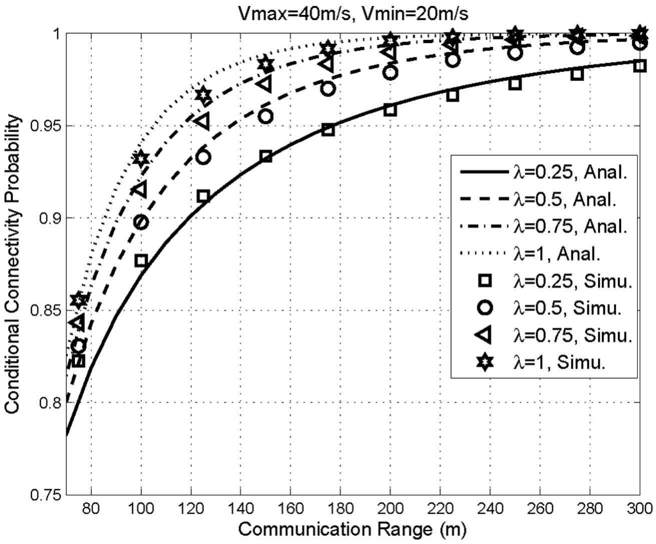
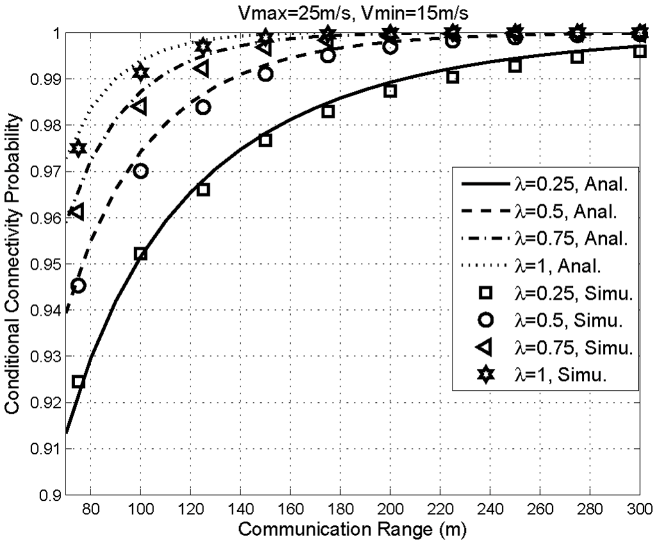


Figure 6 Conditional connectivity probability with various λ when vehicles move with lower speed



4.2 Network connectivity probability

Figure 7 shows both the analytical and simulation results of network connectivity probability P_{net} with varying communication range R and increasing enter density λ, where $v_{\max}$ and $v_{\min}$ are set to 40 and 20 m/s respectively. It is found that the simulation curves are identical with the analytical results. As depicted in Figure 7, P_{net} ascends dramatically with increasing communication range R for $\lambda = 1$. For a fixed network connectivity probability P_{net}, a lower enter intensity λ leads to a larger required communication range R. For example, to obtain a specific network connectivity probability $P_{\mathrm{net}} = 0.9$, the required communication ranges are around 200 m and 350 m for $\lambda = 1$ and $\lambda = 0.5$ respectively; while for $\lambda = 0.25$, the required R is up to 650 m, which almost covers one third of the whole highway segment. This is because slower enter intensity causes further inter-vehicle distance. As a consequence, the required communication range should be large enough to ensure the successful transmission and reception of the vehicular communication.

Figure 7 Network connectivity probability with various λ when vehicles move with higher speed

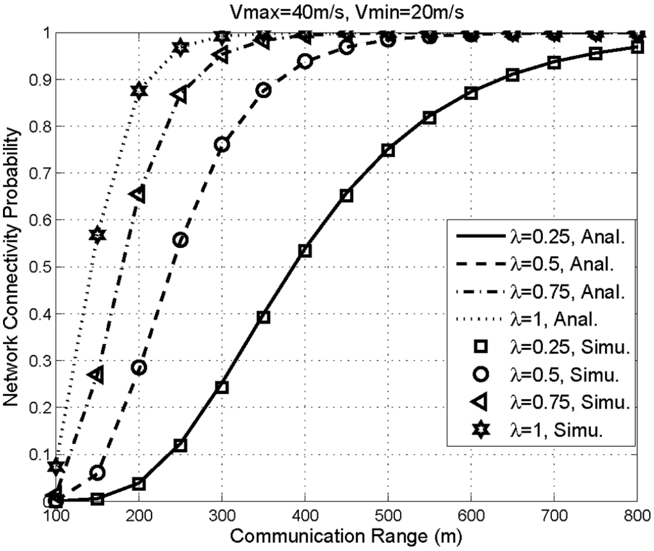
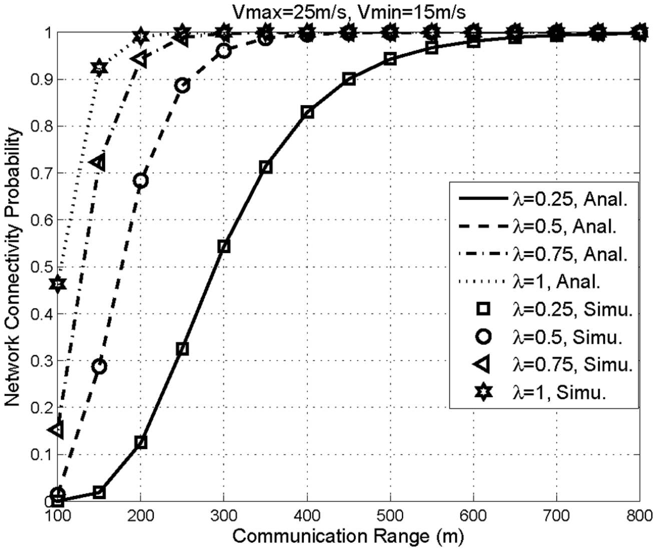


Figure 8 depicts the analytical and simulation results of network connectivity probability when $v_{\max}$ and $v_{\min}$ are fixed at 25 and 15 m/s separately. Evidently, the curves in this figure climb with the same trends as those in Figure 7. The network connectivity probability P_{net} increases with the larger communication range R and the higher enter intensity λ, as well as the slower vehicle speed. Compared to Figure 7, it is obvious that the network connectivity probability ascends slightly faster in this figure, and the required communication range could be less for a specific probability P_{net}. For instance, to attain a fixed network connectivity probability $P_{\mathrm{net}} = 1$, the required communication range should be about 350 m for $\lambda = 1$ in Figure 7, while the required R could be around 250 m which is much shorter with the same λ in Figure 8. This can be explained that the lower mobility results in the closer inter-vehicle distance; therefore, it comes to the conclusion that the vehicles with the slower speed are more possible to maintain the whole network connected.

Figure 8 Network connectivity Probability with various λ when vehicles move with lower speed



4.3 Vehicle isolation probability

Figure 9 reveals both the analytical and simulation results of vehicle isolation probability P_{iso} as a function of communication range R and enter density λ, when $v_{\max}$ and $v_{\min}$ are set to 40 and 20 m/s respectively. It is evident that the vehicle isolation probability degrades rapidly with the increasing communication range and added enter intensity. Apparently, for a fixed communication range, the vehicles with higher enter intensity are less possible to remain at the state of isolation. For instance, when R is 100 m, the probability P_{iso} of $\lambda = 0.25$ is almost 4 times more than the isolation probability of $\lambda = 0.75$. This is because lower enter intensity results in less vehicle density, and probably leads to sparse network and more isolated vehicles.

Figure 9 Vehicle isolation probability with various λ when vehicles move with higher speed

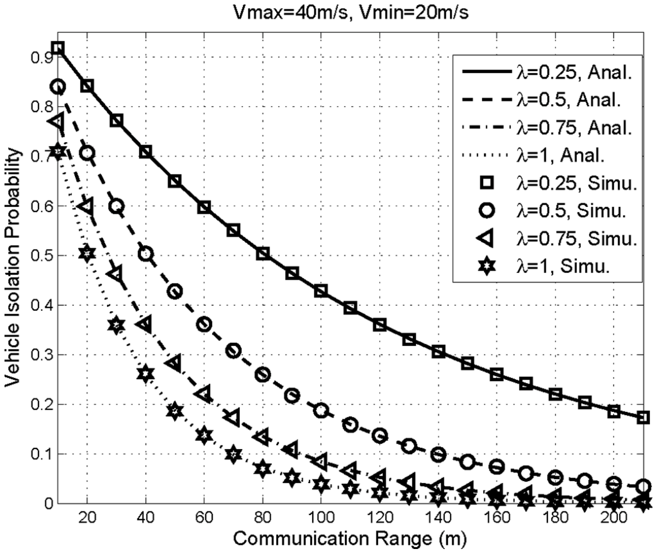
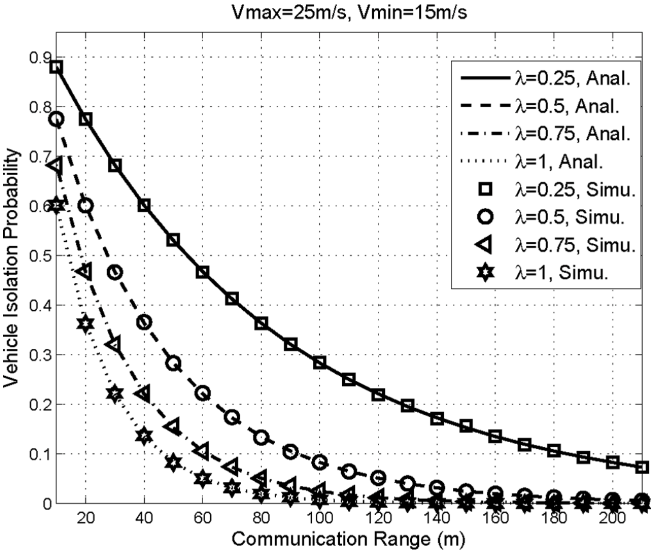


Figure 10 demonstrates the analytical and simulation results of vehicle isolation probability as $v_{\max}$ and $v_{\min}$ are fixed at 25 and 15 m/s separately. Clearly, the trends of the curves in this figure are identical with those in Figure 9, both declining with the increasing communication range and enter intensity. In contrast with Figure 9, the vehicle isolation probability descends more rapidly in terms of lager communication range and added enter intensity in this figure. It comes to the conclusion that the vehicles with slower mobility are less possible to be isolated, especially for those with higher enter intensity and larger communication range. For example, for a specific enter intensity $\lambda = 1$ and the fixed communication range $R = 20$ m, the vehicle isolation probability P_{iso} achieves 0.5 in Figure 9 while the P_{iso} drops to approximately 0.35 in Figure 10. This can be explained that slower vehicle speed leads to closer inter-vehicle distance, and as a consequence there are less isolated vehicles existing in the network.

Figure 10 Vehicle isolation probability with various λ when vehicles move with lower speed



5 Conclusion

In this paper, a simplified yet rational unidirectional highway system model has been built to study the network connectivity performance for VANETs from both the microscopic and macroscopic view. The key factors that have influence on the conditional connectivity probability, global network connectivity, and vehicle isolation probability have been analysed in the paper. By deriving the pdf of the inter-vehicle initial distance and the distribution of vehicles' relative speed, the closed-forms of conditional connectivity probability, network connectivity probability, and vehicle isolation probability are obtained. Finally, the extensive simulation results validate the correctness of our analytical work.

The analytical results of this paper can be applied to reflect the reliability of the inter-vehicle communication. Moreover, the results in the paper are useful for network deployment and can provide the guidance to set appropriate system parameters in order to attain the required connectivity performance. If the network parameter like vehicle enter intensity has already been given, then the drivers can adjust other parameters, such as vehicle speed and communication range, to acquire a specific connectivity probability. Furthermore, reliable routing protocols are required to guarantee the reliability of user service and the safety-related information dissemination. The results in this paper can provide the link connectivity status in accordance with the system parameters and also enable drivers to select reasonable routes on the highway. In the path discovery scheme, the drivers could find the connected paths by selecting the ones with higher connectivity probability. Hence the information could be forwarded through these connected paths to ensure the reliable delivery.

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